

7-4 Factoring $ax^2 + bx + c$

Objective

Factor quadratic trinomials of the form
 $ax^2 + bx + c$.

7-4 Factoring $ax^2 + bx + c$

In the previous lesson you factored trinomials of the form $x^2 + bx + c$. Now you will factor trinomials of the form $ax^2 + bx + c$, where $a \neq 0$.

Check for GCF!

7-4 Factoring $ax^2 + bx + c$

To factor a trinomial like $ax^2 + bx + c$ into its binomial factors, write two sets of parentheses

$$(\quad x + \quad)(\quad x + \quad),$$

Write two numbers that are factors of a next to the x 's and two numbers that are factors of c in the other blanks. Multiply the binomials to see if you are correct.

$$(3x + 2)(2x + 5) = 6x^2 + 19x + 10$$

Handwritten annotations:
- Red circles around $3x$, 2 , $2x$, and 5 .
- Red boxes around 2 and 5 .
- Red boxes around $6x^2$ and 10 .
- Red arrow from 3 and 2 to $6x^2$ with $3 \cdot 2 = 6$ above it.
- Red arrow from 2 and 5 to 10 with $2 \cdot 5 = 10$ above it.
- Green arrow from 2 and 5 to $19x$.

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 1a

Factor each trinomial by **guess and check**.

$$6x^2 + 11x + 3$$

$$\text{GCF} = 1$$

$+bx + c \rightarrow$ same (add)

$$\begin{array}{c} 6 \\ \hline \begin{array}{c} 1 \quad 6 \\ \hline \textcircled{2 \quad 3} \end{array} \end{array} \left(\begin{array}{c} \textcircled{3x+1} \\ \textcircled{2x+3} \end{array} \right) \begin{array}{c} 3 \\ \hline \textcircled{1 \quad 3} \end{array}$$

$$6x^2 + 9x + 2x + 3$$

$$6x^2 + 11x + 3 \checkmark$$

7-4 Factoring $ax^2 + bx + c$

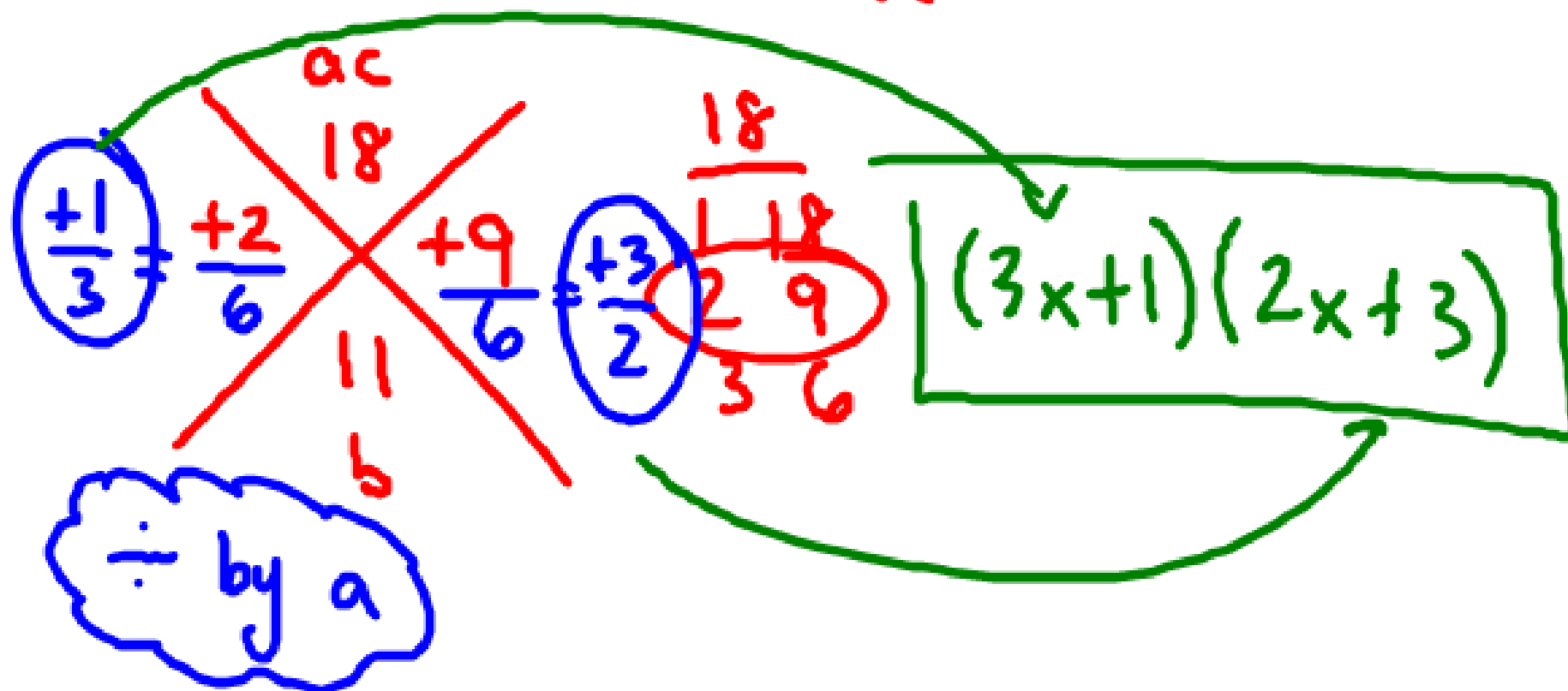
Check It Out! Example 1a

X-Method

Factor each trinomial by ~~guess and check~~.

$$6x^2 + 11x + 3 \quad \text{GCF} = 1$$

$+bx + c \rightarrow$ same (add)



7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 1b

Factor each trinomial by guess and check.

$$3x^2 - 2x - 8 \quad \text{GCF} = 1$$

$-bx - c \rightarrow$ opp (subtract)

↑ Bigger Product

$\frac{3}{1 \ 3}$

$(1x - 2)(3x + 4)$

$\frac{8}{1 \ 8}$
 $\frac{8}{2 \ 4}$

$$3x^2 + 4x - 6x - 8$$

$$3x^2 - 2x - 8 \checkmark$$

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 1b X-Method

Factor each trinomial by ~~guess and check~~.

$$3x^2 - 2x - 8$$

$$\text{GCF} = 1$$

$-bx - c \rightarrow$ opp (subtract)

Bigger #

$$(1x - 2)(3x + 4)$$

7-4 Factoring $ax^2 + bx + c$

So, to factor $ax^2 + bx + c$, check the factors of a and the factors of c in the binomials. The sum of the products of the outer and inner terms should be b .

Product = a

Product = c

$$(\square x + \square)(\square x + \square) = ax^2 + bx + c$$

Sum of outer and inner products = b

O

7-4**Factoring $ax^2 + bx + c$** **Check It Out! Example 2a**

Factor each trinomial. Check your answer.

$$6x^2 + 17x + 5$$

SKIP!

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 2a

Factor each trinomial. Check your answer.

$$6x^2 + 17x + 5$$

SKIP!

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 2b

Factor each trinomial. Check your answer.

$$9x^2 - 15x + 4 \quad \text{GCF} = \underline{1}$$

$-bx + c \rightarrow$ same (add)

$$\begin{array}{c} 9 \\ 1 \quad 9 \\ \hline 3 \quad 3 \end{array} \left((3x - 1)(3x - 4) \right) \begin{array}{c} 4 \\ 1 \quad 4 \\ \hline 2 \quad 2 \end{array}$$

$$9x^2 - 12x - 3x + 4$$

$$9x^2 - 15x + 4 \checkmark$$

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 2b

Factor each trinomial. Check your answer.

$$9x^2 - 15x + 4 \quad \text{GCF} = 1$$

$-bx + c \rightarrow$ same (add)

The diagram illustrates the process of factoring $9x^2 - 15x + 4$ by finding two fractions that multiply to $\frac{4}{9}$ and add to $-\frac{15}{9}$.

Initial attempt: $\left(-\frac{1}{3}\right) \neq \left(-\frac{3}{9}\right)$ and $\left(-\frac{12}{9}\right) = \left(-\frac{4}{3}\right)$. The numbers 36 and 15 are crossed out.

Successful attempt: $\left(-\frac{4}{3}\right)$ and $\left(\frac{3}{9}\right)$. A grid of numbers is shown:

| | |
|---|----|
| 1 | 36 |
| 2 | 18 |
| 3 | 12 |
| 4 | 9 |
| 6 | 6 |

The pair (3, 12) is circled in red, indicating the correct factors.

The final factored form is $(3x-1)(3x-4)$.

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 2c

Factor each trinomial. Check your answer.

$$3x^2 + 13x + 12$$

SKIP!

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 2c

Factor each trinomial. Check your answer.

$$3x^2 + 13x + 12$$

SKIP!

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 3a

Factor each trinomial. Check your answer.

$$6x^2 + 7x - 3 \quad \text{GCF} = 1$$

$+bx - c \rightarrow$ opp (subtract)
↑
Bigger Product

$$\begin{array}{c} \underline{6} \\ 1 \quad 6 \\ \hline 2 \quad 3 \end{array} \quad (3x - 1)(2x + 3) \quad \begin{array}{c} 3 \\ 1 \quad 3 \\ \hline \end{array}$$

$$6x^2 + 9x - 2x - 3$$

$$6x^2 + 7x - 3 \checkmark$$

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 3a

Factor each trinomial. Check your answer.

$$6x^2 + 7x - 3 \quad \text{GCF} = 1$$

$+bx - c \rightarrow$ opp (subtract)
↑
Bigger #

The diagram illustrates the process of factoring $6x^2 + 7x - 3$ using the AC method. It shows two trial factorizations that are crossed out with red lines:

- First trial: $\left(\frac{+3}{2}\right) \left(\frac{+9}{6}\right) \left(\frac{-2}{6}\right) \left(\frac{-1}{3}\right)$. The numbers 18, 7, 18, 2, 9, and 3 are written around the terms. A red arrow points from the 18 above $\frac{+9}{6}$ to the 18 above $\frac{-1}{3}$.
- Second trial: $\left(\frac{+3}{2}\right) \left(\frac{-2}{6}\right) \left(\frac{-1}{3}\right) \left(\frac{+9}{6}\right)$. The numbers 18, 7, 18, 2, 9, and 3 are written around the terms. A red arrow points from the 18 above $\frac{-2}{6}$ to the 18 above $\frac{+9}{6}$.

The correct factorization is shown in a green box: $(2x+3)(3x-1)$. A green arrow points from the 18 above $\frac{+9}{6}$ in the first trial to the 18 above $\frac{-1}{3}$ in the second trial, and another green arrow points from the 18 above $\frac{-1}{3}$ in the second trial to the boxed answer.

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 3b

Factor each trinomial. Check your answer.

$$4n^2 - 1n - 3 \quad \text{GCF} = 1$$

$-bn - c \rightarrow \text{opp (subtract)}$

↑ Bigger Product

$$\frac{4}{1 \ 4}$$
$$\frac{1 \ 4}{2 \ 2}$$

$$(1n - 1)(4n + 3)$$

$$\overset{3}{\textcircled{1 \ 3}}$$

$$4n^2 + 3n - 4n - 3$$

$$4n^2 - 1n - 3 \checkmark$$

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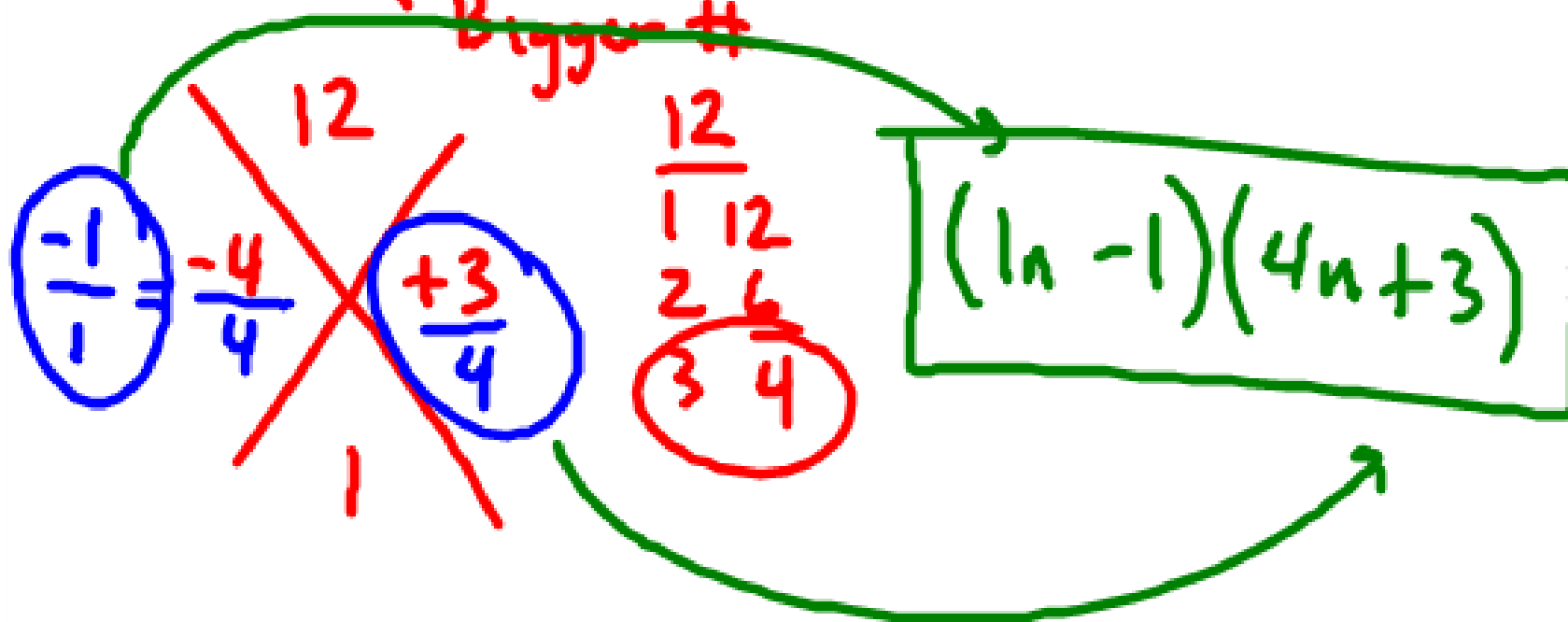
Check It Out! Example 3b

Factor each trinomial. Check your answer.

$$4n^2 - n - 3 \quad \text{GCF} = 1$$

$-bn - c \rightarrow$ opp(subtract)

↑
Bigger #



7-4 Factoring $ax^2 + bx + c$

When the leading coefficient is negative, factor out -1 from each term before using other factoring methods.

Caution

When you factor out -1 in an early step, you must carry it through the rest of the steps.

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 4a

Factor each trinomial.

$$\textcircled{-}6x^2 - 17x - 12 \quad \text{GCF} = -1$$

$$\textcircled{-}1(6x^2 + 17x + 12)$$

$+bx + c \rightarrow$ same (add)

Factor pairs for 72:

| | | | |
|-----------------|-----------------|-----------------|-----------------|
| $\frac{+4}{3}$ | $\frac{+8}{6}$ | $\frac{+9}{6}$ | $\frac{+3}{2}$ |
| $\frac{+12}{6}$ | $\frac{+18}{4}$ | $\frac{+24}{3}$ | $\frac{+36}{2}$ |
| $\frac{+16}{4}$ | $\frac{+18}{4}$ | $\frac{+24}{3}$ | $\frac{+36}{2}$ |
| $\frac{+24}{3}$ | $\frac{+18}{4}$ | $\frac{+12}{6}$ | $\frac{+9}{6}$ |
| $\frac{+36}{2}$ | $\frac{+18}{4}$ | $\frac{+12}{6}$ | $\frac{+9}{6}$ |
| $\frac{+72}{1}$ | $\frac{+18}{4}$ | $\frac{+12}{6}$ | $\frac{+9}{6}$ |

Correct pair: $\frac{+3}{2}$ and $\frac{+9}{6}$

$$\textcircled{-}1(3x + 4)(2x + 3)$$

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 4a

Factor each trinomial.

$$-6x^2 - 17x - 12$$

SKIP!

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 4b

Factor each trinomial.

$-3x^2 - 17x - 10$ GCF = -1

$-1(3x^2 + 17x + 10)$

$+bx + c \rightarrow$ same (add)

$\frac{+2}{3}$ $\frac{+15}{3}$ $\frac{+5}{1}$ $\frac{+30}{15}$

30

17

$-1(3x+2)(1x+5)$

7-4 Factoring $ax^2 + bx + c$

Check It Out! Example 4b

Factor each trinomial.

$$-3x^2 - 17x - 10$$

SKIP!

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HOMEWORK

BASIC: p. 484 #25-64, 68-74, 76

AVERAGE: p. 484 #25-70, 75-76

HOMEWORK HELP?

#30, 35, 48, 50, 58, 64

not # 49-51, 60

Inclass